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A Modified Alamouti Scheme for Frequency Selective Channels Incorporating Turbo Equalization

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Abstract—The Alamouti scheme can achieve transmit diversity in the presence of frequency flat fading channels. The technique has received considerable attention mainly because of its processing simplicity. Recently, a modification to the Alamouti scheme to achieve transmit diversity in the presence of frequency selective fading channels was proposed. This modified scheme requires parallel equalization of the transmitted information blocks. In this paper a Turbo equalization system is proposed that requires the same number of component equalizers but offers improved system BER performance. The idea treats a pair of frequency selective channels as a parallel-concatenated Turbo encoder.

I. Introduction

Since the introduction of Space-Time (S-T) codes, the great majority of the schemes adopt the assumption of frequency flat radio channels. In practice, the increasing demand for high-speed applications leads to the design of wideband systems, which in general require some form of equalizing system at the receiver. The efficient adaptation of S-T coding schemes for practical wideband systems is still an area of great interest.

A good example of such adaptation was given recently in [2] for the transmit diversity scheme proposed by Alamouti in [1]. As elaborated in section II, conceptually the original (narrowband) and adapted (wideband) schemes are very similar. However the two differ distinctively in two ways; the orthogonal transmit matrix in the latter is constructed by information symbol vectors (rather than discrete information symbols) and on the receiver side the latter requires separate equalization of the decoupled-maximally ratio combined received sequences.

Following from the concept of Turbo Equalization as introduced in [4] (and explored further in [5], [6]) the structure of the wideband transmit diversity system has been exploited to develop a new iterative equalization scheme. Perhaps the most attractive feature of the new scheme is that it can achieve remarkable performance while relying entirely on the two equalizers that need to be present, in some form, even if the proposed Turbo scheme is not used.

II. Transmit Diversity Scheme for ISI Channels

The idea behind the proposed system is to try and create an equalization analogue of the classical Parallel Concatenated Turbo (PCT) encoder/decoder. In this analogue, the role of the encoders is taken by the Inter Symbol Interference (ISI) channels, that of the bit-wise interleaver by a symbol-wise interleaver and two equalizers capable of accepting a priori information substitute the two component decoders in the model. A suitable architecture for constructing this analogue is achieved by a transmit diversity scheme in the presence of ISI channels. In this section a description of this architecture is given. The approach differs to the one given in [2].

A. Narrowband Transmit Diversity Scheme

Let us recall the basics of the narrowband transmit diversity scheme. The basic idea is to construct an orthogonal transmit matrix:

$$\underline{\underline{X}} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \quad (1)$$

where x_1 and x_2 are information symbols in the transmission sequence chosen from some finite set (e.g. QPSK constellation) and the operator $(.)^*$ signifies complex conjugation. The columns of $\underline{\underline{X}}$ correspond to two consecutive symbol periods and the rows to the two transmitting antennas. Note that the inner product between the rows of $\underline{\underline{X}}$ is zero, as this is the basis for it to be orthogonal.

In the case of narrowband channels the propagation paths between the two transmit antennas and the receiver can be modelled by two complex factors: $h_1 = |h_1| \cdot e^{j\theta_1}$ and $h_2 = |h_2| \cdot e^{j\theta_2}$, where the time dependency of the two is not indicated for simplicity. Assuming that these remain constant over two consecutive symbol periods, then the received symbols (in these two symbol periods) are given by:

$$r_1 = h_1 x_1 + h_2 x_2 + n_1 \quad (2a)$$

$$r_2 = -h_1 x_2^* + h_2 x_1^* + n_2 \quad (2b)$$

where n_1 and n_2 are independent Gaussian distributed noise samples with zero mean and variance $\sigma^2/2$ per dimension, where σ^2 equals the noise power. Assuming knowledge of the Channel State Information (CSI), a maximum ratio combiner can be formed at the receiver, which at the same time exploits the orthogonality of \underline{X} in order to decouple x_1 and x_2 in (2a) and (2b):

$$\begin{aligned} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} &= \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \cdot \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \cdot \begin{bmatrix} h_1 x_1 + h_2 x_2 + n_1 \\ -h_1^* x_2 + h_2^* x_1 + n_2^* \end{bmatrix} = \\ &= \begin{bmatrix} (|h_1|^2 + |h_2|^2)x_1 + h_1^* n_1 + h_2 n_2^* \\ (|h_1|^2 + |h_2|^2)x_2 - h_1 n_2^* + h_2^* n_1 \end{bmatrix} \end{aligned} \quad (3)$$

This allows the transmitted symbols to be estimated by applying the maximum likelihood rule on s_1 and s_2 separately.

B. ISI Channel Case

For a wideband system, delayed components of the transmit symbol sequence spread into adjacent symbols' intervals thus leading to ISI. The channels in this case can no longer be modelled by single complex time-varying factors. Instead, it is typical to model the propagation paths as the impulse responses of Finite Impulse Response (FIR) filters, whose order will depend on the delay-spread of the environment and the rate at which symbols are transmitted.

$$\underline{h}_1 = \sum_{m=1}^M h_1^m \cdot \delta[k-m] \quad (4a)$$

$$\underline{h}_2 = \sum_{m=1}^M h_2^m \cdot \delta[k-m] \quad (4b)$$

where $M-1$ is the memory order of the two channels, k is a discrete time delay variable taking values in multiples of the symbol period, and $\delta[k] = \begin{cases} 1, & k=0 \\ 0, & k \neq 0 \end{cases}$. Again time dependency is

not illustrated. Because of the ISI introduced by the channel into the data stream, the adapted scheme is based on symbol vectors rather than discrete symbols. In a practical system these symbol vectors might correspond to information packets, or parts of them. In any case their length will need to be constrained according to the coherence time of the propagation environment. Edge effects between consecutive transmitted vectors can be avoided by allowing guard time intervals between them or handled by placing training information on the ends of the vectors as suggested in [2]. In the modified transmit diversity scheme the orthogonal transmit matrix will be constructed as:

$$\underline{X} = \begin{bmatrix} \underline{x}_1 & -\underline{x}_2^* \\ \underline{x}_2 & \underline{x}_1^* \end{bmatrix} \quad (5)$$

where $\underline{x}_1 = [x_1^1, x_1^2, \dots, x_1^N]$, $\underline{x}_2 = [x_2^1, x_2^2, \dots, x_2^N]$, where N is the length of the sequence. The complex conjugation of a time sequence is achieved by conjugating all the coefficients in the sequence and also by time reversing the whole sequence: $\underline{x}_i^* = [(x_i^N)^*, (x_i^{N-1})^*, \dots, (x_i^1)^*]$. In order to express the received signals we could use expressions similar to (2a) and (2b) involving the new vector variables and substituting multiplications by discrete convolutions. Instead of this representation, we will use the channel filtering matrices \underline{H}_1 and \underline{H}_2 which explicitly describe the linear transformations that the data undergo as it is filtered through the channels. These two matrices are defined according to:

$$\underline{H}_i = \begin{bmatrix} \underline{h}_i^T & 0 & . & 0 \\ 0 & \underline{h}_i^T & 0 & 0 \\ . & 0 & . & 0 \\ 0 & 0 & 0 & \underline{h}_i^T \end{bmatrix} \quad (6)$$

for some channel \underline{h}_i and their size is $N \times N$. The received signals over two data vector periods can be expressed as:

$$\underline{r}_1 = \underline{H}_1 \underline{x}_1^T + \underline{H}_2 \underline{x}_2^T + \underline{n}_1^T \quad (7a)$$

$$\underline{r}_2 = -\underline{H}_1 (\underline{x}_2^*)^T + \underline{H}_2 (\underline{x}_1^*)^T + \underline{n}_2^T \quad (7b)$$

where $(.)^T$ is the transposition operator and \underline{n}_1 and \underline{n}_2 represent a sequence of noise samples having length N . Similarly to the ISI free case, using knowledge of the CSI a maximum ratio combiner can be formed that is also able to decouple the two transmitted sequences:

$$\begin{aligned} \begin{bmatrix} \underline{s}_1 \\ \underline{s}_2 \end{bmatrix} &= \begin{bmatrix} \underline{H}_1^* & \underline{H}_2 \\ \underline{H}_2^* & -\underline{H}_1 \end{bmatrix} \cdot \begin{bmatrix} \underline{r}_1 \\ \underline{r}_2 \end{bmatrix} = \\ &= \begin{bmatrix} (\underline{H}_1^* \underline{H}_1 + \underline{H}_2 \underline{H}_2^*) \underline{x}_1^T + \underline{H}_1^* \underline{n}_1^T + \underline{H}_2 (\underline{n}_2^*)^T \\ (\underline{H}_2 \underline{H}_1^* + \underline{H}_1 \underline{H}_2) \underline{x}_2^T - \underline{H}_1 (\underline{n}_2^*)^T + \underline{H}_2^* \underline{n}_1^T \end{bmatrix} \end{aligned} \quad (8)$$

Comparing (8) to (3) enables the direct analogy between the original and the adapted transmit diversity systems to be observed. It should be stressed that complex conjugation of a time sequence requires time reversal of the sequence as well as conjugation of the individual coefficients. Conjugation of a channel filtering matrix represents conjugation of each of the channel vectors in (6). By this kind of processing, a full transmit diversity gain can be achieved as in the narrowband case [7].

Equation (8) shows that effectively \underline{x}_1 and \underline{x}_2 are filtered through the same channel:

$$\begin{aligned} \underline{a} &= \sum_{m=-M}^M a[m] \cdot \delta(k-m) = \\ &= \sum_{m=-M}^M \{ \rho_{\underline{h}_1 \underline{h}_1} [m] + \rho_{\underline{h}_2 \underline{h}_2} [m] \} \cdot \delta(k-m) \end{aligned} \quad (9)$$

but in infinite spatial separation, where $\rho_{hh}[m]$ is the m -lagged autocorrelation function of vector \underline{h} . This allows \underline{s}_1 and \underline{s}_2 to be equalized separately in order to produce estimates for \underline{x}_1 and \underline{x}_2 . Figure 1 illustrates a block diagram of the modified transmit diversity scheme together with its equivalent representation, which is two Single Input Single Output (SISO) wideband systems.

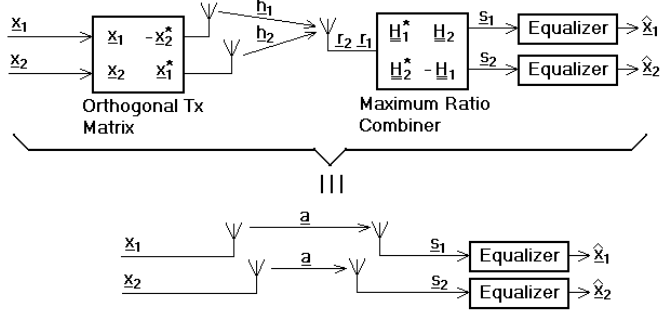


Figure 1: Transmit Diversity Scheme for frequency selective channels and its equivalent representation

The rows of the matrix in the second line of (8) show that the linear processing at the receiver yields the matched filter's output, as if we had applied matched filtering in a receive diversity system and combined the two branches. It is observed that this processing introduces temporal correlation in the elements of the noise vectors thus preventing optimal detection using a Viterbi or a Maximum A-Posteriori (MAP) algorithm. In addition, noise whitening is not possible because the two independent noise vectors are filtered with a different impulse response. However simulation results (given in section IV.) suggest that in practice this fact does not have a dramatic effect on the performance of a MAP based equalizer.

III. Turbo Equalization Scheme

An examination of the structure of the modified transmit diversity scheme shows that it bears strong resemblance to that of a PCT encoder/decoder. Indeed, the separated channels can be used as the constituent encoders in a Turbo encoder while the two equalizers can be easily adapted to take a role similar to that of a Turbo decoder. However, there are two important elements missing in order to completely emulate the structure of a PCT encoder; the interleaver and the recursive nature of the channels.

A. Symbol-Wise Interleaver

The first requirement can be easily satisfied, by pseudo-randomly interleaving the elements of \underline{x}_1 so that \underline{x}_2 does not represent a different information vector but a symbol-wise interleaved version of \underline{x}_1 . Obviously this modification results in halving the rate of the transmit diversity system. Similar to the bit-wise interleaver in a PCT encoder, the purpose of the

symbol-wise interleaver is to decorrelate the inputs to the two channels so that the probability of both \underline{s}_1 and \underline{s}_2 having small squared Euclidian distance¹ to some other 'codeword' is diminished. Of course, the random variations of the channel coefficients over time make the distance properties of the 'code' random, as well.

B. Recursive Encoders

A key aspect of Turbo Codes is the recursive nature of the constituent convolutional encoders. An important property of recursive encoders is that they make any weight-1 input sequence non-divisible with the feedback polynomial of the encoder giving infinite weight codewords (associated to weight-1 inputs). It has been shown in [3] that this property greatly influences the average probability of bit error in Turbo Codes.

Returning to the Turbo equalization scheme, if the propagation channels were in some manner made recursive this would help to diminish the probability of having a small squared Euclidian distance between any pair of codewords that are associated to weight-1² inputs. This is justified because, similar to a typical recursive encoder, the same conditions would apply for the divisibility of the input sequence by the feedback polynomial in a recursive channel. Simulation results presented in section IV. show that recursive channels are necessary if sustained iterative improvement is to be achieved.

Using the standard polynomial representation for describing convolutional encoders in order to express the output of the propagation channel, we can write:

$$s_i(D) = x_i(D) \cdot a(D) \quad (10)$$

where D is the unit delay variable. Equation (10) is an alternative representation of the discrete convolution of the information vector with the channel's response. We are looking for a channel that has a feedback polynomial in addition to the feedforward polynomial. Instead of (10), we would like the channels to behave according to:

$$s_i(D) = x_i(D) \cdot \left[\frac{a(D)}{g(D)} \right] \quad (11)$$

where $g(D)$ is the feedback polynomial describing the recursive structure of the channel. Weighting coefficients in

¹ If we think of the propagation channels as convolutional encoders, because the channel coefficients and the operations of multiplication and addition belong to the infinite field of complex numbers, we can no longer talk about the Hamming distance between two 'codewords', i.e. two received sequences. A suitable metric for this case would be the squared Euclidian

distance between two 'codewords': $d^E(\underline{s}^m, \underline{s}^n) = \sum_{k=1}^{N+M-1} |s^m[k] - s^n[k]|^2$

² At symbol level a weight-1 sequence is one that has exactly one symbol difference from the 'all zero' sequence.

$g(D)$ as well as multiplication and addition should be defined in some extension field depending on the type of modulation used (e.g. $GF(2)$ for BPSK, $GF(2^2)$ for QPSK, etc.). Of course, it is physically impossible to have a recursive channel but we can re-express (11) as:

$$s(D) = \left(\frac{x(D)}{g(D)} \right) \cdot a(D) = y(D) \cdot a(D) \quad (12)$$

which suggests that the forward and recursive parts of the filter can be separated. Figure 2 illustrates an example of how a recursive filter could be practically separated into a purely recursive and a purely non-recursive section.

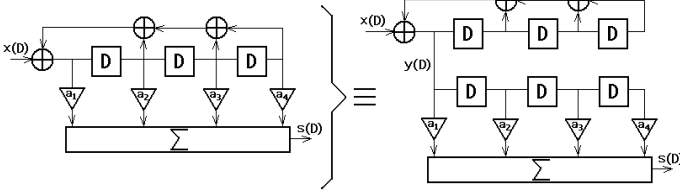


Figure 2: Recursive filter and equivalent construction with separated feedback and feedforward filters

Although additional memory elements need to be utilized in order to realize the recursive structure, this has no effect on the effective memory order of the channel, which is still $M-1$. Hence the complexity of a trellis based equalizer does not increase by making the channels recursive.

C. Turbo Equalizer

Exploiting the structure of the system depicted in Figure 1, an iterative equalization scheme can be realised by using suitable component equalizers (able of accepting a-priori information for each transmitted symbol) and interconnecting them in a classical Turbo decoding fashion in order to allow extrinsic information exchange. For a K -state modulation scheme, a MAP equalizer calculates the a-posteriori probabilities for each symbol in the transmit information sequence according to:

$$\Pr(x_i = j | \underline{s}) = \sum_{\forall (l' \rightarrow l)} [\alpha_{i-1}(l') \cdot \gamma_i^j(l', l) \cdot \beta_i(l)] / \Pr(\underline{s}) \quad (13)$$

where

$$\gamma_i^j(l', l) = \Pr(x_i = j) \cdot \exp\left(-\frac{1}{2 \cdot \sigma^2 \cdot (\sum_m |h_1^m|^2 + |h_2^m|^2)} \cdot |s_i - (a * y_{(i-M) \rightarrow i})|^2\right) \quad (14)$$

$$\alpha_i(l) = \sum_{l'=0}^{2^{M-1}-1} \alpha_{i-1}(l') \cdot \sum_{j=0}^{K-1} \gamma_i^j(l', l) \quad (15)$$

$$\beta_i(l) = \sum_{l'=0}^{2^{M-1}-1} \beta_{i+1}(l') \cdot \sum_{j=0}^{K-1} \gamma_{i+1}^j(l', l) \quad (16)$$

$\Pr(x_i = j)$ is the a-priori probability for each transmitted symbol previously calculated, the index in the summation in

(13) signifies for all state transitions ($l' \rightarrow l$) at time i caused by symbol j and $*$ denotes discrete convolution. Note also that in (14) the noise variance is scaled by the energy in the two channels because of the linear processing that takes place at the receiver (see (8)). Substituting (14) in (13), taking $\Pr(x_i = j)$ out of the summation and collecting the terms left in the summation, it follows that:

$$\Pr(x_i = j | \underline{s}) = \Pr(x_i = j) \cdot \tilde{\Pr}(x_i = j) / \Pr(\underline{s}) \quad (17)$$

$\tilde{\Pr}(x_i = j)$ is the quantity which, after (de-)interleaving, will serve as the new a-priori information in the next Turbo iteration. In order to calculate $\tilde{\Pr}(x_i = j)$ we can use the fact that $\sum_{j=0}^{K-1} \tilde{\Pr}(x_i = j) = 1$ and also calculate $K-1$ probability

ratios: $\frac{\tilde{\Pr}(x_i = K-1)}{\tilde{\Pr}(x_i = 0)}, \frac{\tilde{\Pr}(x_i = K-2)}{\tilde{\Pr}(x_i = 0)}, \dots, \frac{\tilde{\Pr}(x_i = 1)}{\tilde{\Pr}(x_i = 0)}$ based on (17).

The iterative procedure can be terminated, by finding the maximum a-posteriori probability for each symbol and making a hard decision. Figure 3 illustrates the proposed iterative equalization scheme.

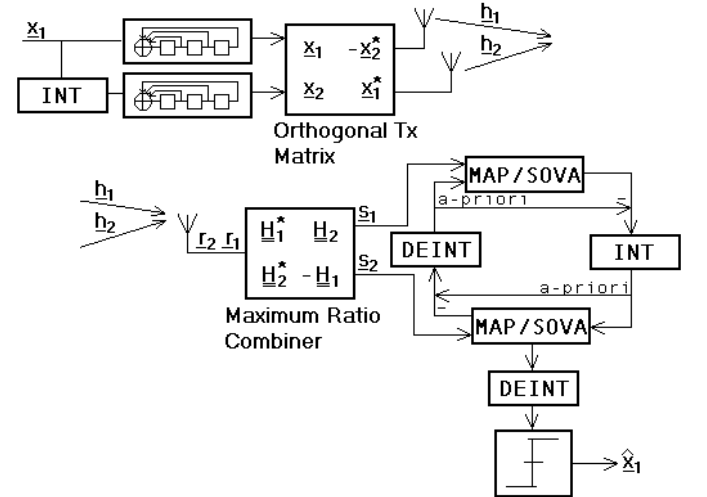


Figure 3: Proposed iterative equalization scheme

IV. Simulation Results

The proposed system has been simulated in order to produce Bit Error Rate (BER) and Frame Error Rate (FER) curves versus Signal to Noise Ratio (SNR). Slow³ and uncorrelated Rayleigh fading channels with memory order 2 (3-taps) have been assumed. The energy in the channels is normalised so that $E[\sum_{m=1}^3 |h^m|^2] = 1$, where $E[\cdot]$ is the statistical expectation operator. A rectangular power delay profile has been imposed

³ In this case by the term slow we imply that the channel state remains constant over two consecutive information block periods.

on the channels. The memory order of the ‘effective’ channels (as given in (9)) is therefore 4 (5-taps). BPSK has been used for modulating the binary data, which are grouped in frames of 1012 bits. To each modulated block an additional 12 symbols have been added for initialising and terminating both channel trellises. A total of 4096 BPSK symbols are transmitted through the two antennas for each frame. The pseudo random symbol-wise interleaver is of size 1016 as 4 additional trellis terminating symbols for the first recursive encoder need to be interleaved. The recursive encoders have been chosen to match the memory order of the ‘effective’ channels, i.e. 4. The recursive filters used are described by the polynomial: $g(D)=1+D+D^3$. The system has also been simulated without the recursive filters in order to illustrate their importance in terms of achieving significant performance improvement. On the receiver side a single antenna is utilised and perfect knowledge of the CSI is assumed.

When recursive filtering is used a significant BER performance improvement, as the number of iterations increase, can be observed. This gain increases at lower error rates, reaching 7.3 dB at a BER of 10^{-5} . The FER curves show an almost constant gain when $10^{-1} \leq \text{FER} \leq 10^{-4}$ (after 10 iterations) of about 9.1 dB . In terms of BER, the results imply that the system can achieve significant iterative improvement provided that not both of the propagation channels are in deep fades (i.e. both very poor in energy). We expect that systems with more diversity (e.g. higher order propagation channels or using receive diversity as well) will be better exploited by the proposed iterative scheme, yielding more performance gain. It is worth noticing the behavior of the system without recursive filtering. In this case no significant gain is achieved after the first iteration.

V. Conclusions

A novel iterative equalization scheme, based on the architecture of a wideband transmit diversity system, has been proposed. The key parameter for achieving sustained iterative improvement is the recursive nature of the propagation channels that has been introduced artificially by pre-filtering the information sequence in a symbol level. The iterative equalization process is based on two MAP/SOVA equalizers interconnected in parallel TURBO decoding fashion. The scheme can achieve substantial performance gain but at the expense of information throughput, which is halved, and an increase in equalizer complexity. The scheme as it stands, is thus more suitable for wideband applications where power efficiency and/or transmission reliability is of greater importance than system complexity.

VI. Acknowledgements

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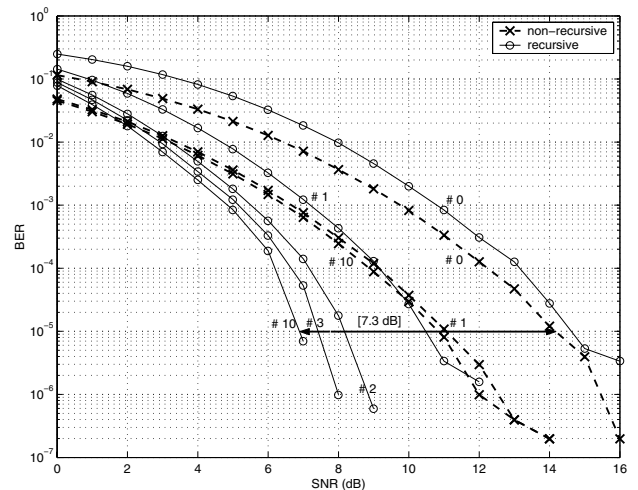


Figure 4: BER performance of iterative equalization scheme with and without recursive filtering

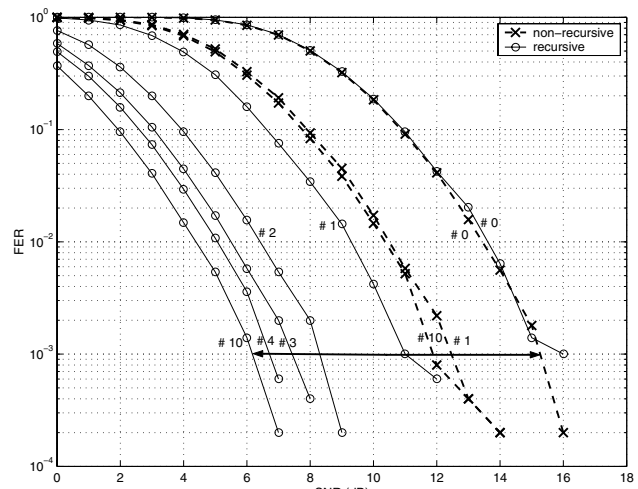


Figure 5: FER performance of iterative equalization scheme with and without recursive filtering

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